Analysis of Bandwidth Requirement of Users in Flexible Reuse Cellular Networks

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Abstract—In flexible frequency reuse scheme namely fractional frequency reuse (FFR), the total frequency resource is divided logically into cell center and cell edge user bands. However, while frequency resource is partitioned into two segments, the amount of average bandwidth required by cell center and cell edge users is quite different and influences the system performance and design. We developed the analytical approach to evaluate the average bandwidth required by cell center and cell edge users for real-time (RT) traffic. It is seen that the analytical results are closely matched with the simulations and indicate that the model is a good approximation for bandwidth requirement of users in a cellular environment.

Index Terms—LTE, WiMAX, fractional frequency reuse, SINR threshold, average bandwidth, real-time traffic.

I. INTRODUCTION

ORTHOGONAL Frequency Division Multiple Access (OFDMA) based 4G cellular networks [1] are designed to support a large number of voice and video traffic users. Such high capacity is achieved by use of unity frequency reuse networks. However, in unity reuse systems, cell edge users experience heavy interference from neighboring cells which causes Quality of Service (QoS) difference for cell center and cell edge users. To improve the cell edge user performance, Fractional Frequency Reuse (FFR) is proposed [2]. In FFR, the cell area is divided into two parts. Cell center region is called reuse one band and edge region is called reuse three band. The edge band of a cell is partitioned into three subbands and are used at the edge regions of the adjacent cells [3] in a reuse three fashion.

It is worthwhile to consider that when the User Equipments (UEs) are in cell center or cell edge region, it is important and useful to calculate the amount of resource (bandwidth) being used by those UEs. Normally, the bandwidth required by a User Equipment (UE) is dependent on its location influenced by SINR and reuse factor of the system. On an average, users need more bandwidth to maintain the same data rate as they move away from cell center. Generally reuse 3 scheme requires lesser bandwidth than reuse 1 scheme especially at edge regions. This difference in using the resources by the UEs in a cell influences the performance of a cellular system. Moreover, in flexible reuse schemes, when the bandwidth is partitioned between cell center and cell edge, it is required to find the amount of bandwidth being reserved to the center and edge regions. However, if the amount of resources required by UEs is known one can find the system capacity in terms of the number of UEs supported in a cell and the amount of bandwidth being reserved for cell center and edge band. The authors in [3] presented the impact of SINR threshold $\gamma_{th}$ and bandwidth ratio on the performance at cell edge and the whole cell. Different bandwidth partitioning methods for Real Time (RT) service are proposed in [3]. There they have shown that the $\alpha$ changes with $\gamma_{th}$. However, such a real time analysis of FFR system needs the average bandwidth utilization of UEs while they are in any particular band of a cell. In this work, the goal is to find the amount of average bandwidth required by cell center and cell edge UEs in a cellular system. While evaluating the average bandwidth required by cell edge UEs, it is important and required to find the upper limit of SINR for cell center and cell edge regions. In the existing literature [4], [5], UEs in a cell are partitioned into cell center and cell edge regions by considering the fixed $\gamma_{th}$. While the UEs are separated into cell center and edge regions, these works did not focus on how much amount of bandwidth the UEs require while they are in any of the corresponding locations. This paper presents the analytical framework to evaluate the amount of bandwidth required by the UEs in a cell.

II. SYSTEM MODEL AND ASSUMPTIONS

We consider OFDMA based downlink cellular networks with Urban Micro (UMi) scenario as shown in Fig. 1. We use hexagonal 19 cells layout with inter site distance (ISD) of $\sqrt{3} R$. UEs are distributed in an uniform random fashion in a circle with radius $R$. The location of UE ‘$u$’ is specified by $(r, \theta)$ with respect to the transmitting base station located at the center of the cell $(0,0)$, where $0 \leq r \leq R$ and $0 < \theta < 2\pi$. The SINR experienced by user $u$ is given by

$$\gamma_{u,b}(r, \theta) = \frac{P_{r_b}(r, \theta)}{\sum_{i \in I(b)} P_{r_i}(r, \theta) + P_N}$$

where $P_N$ is the noise power, $P_{r_i}(r, \theta)$ is the power received from the $i^{th}$ base station. The suffix $b$ takes the value within $\{c, e\}$, where $c$ and $e$ indicate center band and edge band, respectively. The value of $i=0$ indicates the signal from the desired base station. The set $\{I(b)\}$ is the index of base stations which cause interference in $b^{th}$ band. The received power by user $u$ from the $i^{th}$ base station is given by

$$P_{r_i}(r, \theta) = P_{Ti} L d_{u_i}^{-\eta_p} |x_{u,i}|^2,$$

where the transmit power from $i^{th}$ base station is $P_{Ti}$, $x_{u,i}$ is the shadowing component which is lognormal distributed, $|h_{u,i}|^2$ is the power of Rayleigh fading component follows Gamma distribution with unity mean, $n_p$ is the pathloss exponent, and $L$ includes fixed loss.
Let the total number of subcarriers available be $N_{sc}$. The received base stations are modeled as lognormal RV having power from all the base stations at a point is approximated $\text{SINR}$ distribution analytically. The received total interference are assumed to follow lognormal distribution while evaluating $\text{SINR}$ threshold $(\gamma)$. Therefore $\gamma$ is a scaling constant. $\text{SINR}$ threshold $\gamma$ is used to classify the UE's between center and edge category. If $\gamma_{c,e}(r, \theta) \geq \gamma$ the UE is classified as center user otherwise it is a edge user. That is, a UE is allocated in band $b = 'c'$ if $\gamma_{c,e}(r, \theta) \geq \gamma$, and $\gamma_{c,e}(r, \theta)$ is the mean wideband $\text{SINR}$ of a UE when in center band region. Let the effective bit rate required by a user be denoted by $R^e_b$ (in Kbps). The bandwidth (in Hz) needed to support the bit rate requirement to guarantee delivery of a RT service [9] by considering semi persistent scheduling of RT traffic while fulfilling the QoS requirements (delay) are denoted as

$$b_u(r, \theta) = \frac{R^e_b}{\beta \log_2(1 + \frac{\gamma_{c,e}(r, \theta)}{1 - \eta})}.$$  

where $\gamma_{u,b}(r, \theta)$ is the mean wideband $\text{SINR}$ of a UE at a location $(r, \theta)$ when in band $b$, $b_u(r, \theta)$ is the required bandwidth to support the rate $R^e_b$ at location $(r, \theta)$ which experience the $\text{SINR}$ $\gamma_{u,b}(r, \theta)$, $\beta$ and $\eta$ are system implementation losses [10] and these are in ranges $0 < \beta < 1$, $\eta > 1$, and we use $\beta = 0.83$ and $\eta = 4$ dB in accordance with LTE.

In an OFDMA system let the bandwidth available be $B$. Let the total number of subcarriers available be $N_{sc}$. Therefore the subcarrier bandwidth $\Delta f_{sc} = \frac{B}{N_{sc}}$, where $f_s$ is the over-sampling factor. The minimum rate requirement of user which is equal to 8.4 Kbps is assumed for Voice over IP (VoIP). Shadowing is assumed to be uncorrelated and user radius is limited to cell radius instead of considering handoff mechanism for analytical simplicity.

### B. Analytical Model for Average Bandwidth of Users

It is assumed that the bandwidth required by a UE, who needs a minimum guaranteed rate, depends only on $\text{SINR}$ experienced by that UE. Since $\text{SINR}$ varies from location to location in a cell, per UE bandwidth varies at different locations. For RT service, every UE has to get a minimum specified rate. In the scenario considered here, all UEs have fixed minimum specified rate $R_u$. We assume that all UEs served in the system are provided exactly their minimum specified rate, and if the system fails to satisfy this criteria then that UE is blocked. Since the rate is fixed, we get the relation between user bandwidth and $\text{SINR}$ by substituting $R^e_b = R_u$ in (4). The average bandwidth required by a UE in center band ($\bar{B}_{A_c}$) and edge band ($\bar{B}_{A_e}$) are given as

$$\bar{B}_{A_c} = \int_{\gamma} \int_{\gamma} \frac{1}{\pi R^2} \int_{\gamma} b_u(r, \theta) P(\gamma | f) \gamma r dr d\theta,$nolabel \]  

and

$$\bar{B}_{A_e} = \int_{\gamma} \int_{\gamma} \frac{1}{\pi R^2} \int_{\gamma} b_u(r, \theta) P(\gamma | f) \gamma r dr d\theta.$$  

In (5) and (6), the term $P(\gamma | f) \gamma$ is the Probability Density Function (PDF) of $\gamma$ conditioned on $f = \gamma$ and the term $P(\gamma | f) \gamma$ is the PDF of $\gamma$ conditioned on $f \leq \gamma$. In order to evaluate (5) and (6), it is required to find the exact lower and upper limits of $\text{SINR}$, $\gamma$. For center band region, the lower and upper limits of $\text{SINR}$ are $\gamma$ and $\gamma_{max,c}$ (maximum value of center band $\text{SINR}$) respectively. These can be estimated easily, because the upper limit is the $\text{SINR}$ nearest to base station and the lower limit is the $\gamma$. Moreover, it is also essential to find the lower and upper limits of $\text{SINR}$ while evaluating bandwidth required by edge band users. The lowest possible limit in edge band can be assumed to be zero. But the upper limit, the maximum value of $\text{SINR}$ at the edge band $\gamma_{max,e}$ is difficult to estimate. This upper limit is the value of $\gamma$ at edge band, at the point, where the $\gamma$ at center band is equal to $\gamma$ if only pathloss is considered. But due to shadowing a deterministic estimation of $\gamma$ for edge band is difficult. This is because of changing in $\gamma$ due to the varying channel condition. The $\gamma$ is determined by instantaneous channel quality and interference power. The instantaneous interference information may not be available at the base station. Moreover, the transmission decision may affect the interference level in one band, which may affect the transmission decision of other base stations. Furthermore, while the users move between center and edge bands of a cell, the interference pattern will change. Due to this change in interference, $\gamma_{max,e}$ varies. Hence we need a maximum limit for edge band $\text{SINR}$ in order to evaluate (6). Therefore, we propose an analytical model for the upper limit of edge band $\text{SINR}$ as follows. In this letter, it is to be noted that the terms user and UE are used interchangeably.

### III. PROPOSED ANALYTICAL MODEL FOR MAXIMUM EDGE BAND $\text{SINR}$

To determine the maximum edge band $\text{SINR}$, initially, we evaluate the $\gamma_{th}$ which is a lower limit for center band. By
assuming both the received and interference signal powers to be log-normally distributed, we derive a theoretical approximation for upper limit of edge band SINR. The following assumption is made regarding the location of a user in the cell where it experiences maximum edge band SINR. That is, center and edge band users experience same average received power at all locations. The factor which improves reuse three SINR is the difference between reuse one and reuse three interferences. It is based on geometrical consideration of the base stations having omni-directional antennas as shown in Fig. 2. In the figure there are two contours, i.e., reuse one (indicated with red color) and reuse three (indicated with orange color) contours. While averaged over all the cells, these two drawn circles will give equal average received power (or, average received power contours). From the geometry, we assume that the edge user boundary to be circular/hexagonal. Therefore, if a UE moves from the base station in a 30 deg direction, the distance to the nearest interfering base station is geographically maximum for edge user which is marked by red circle. Thus the average interference is minimum at 30 deg. Hence, reuse three interference power will be minimum at 30 deg for contours of equal received power. The users coming in the red circle have same average received power. Hence, on an average the SINR will be maximum. The mean and standard deviation of reuse one and reuse three received and interference powers at this location are obtained by using F-W method [6].

Through analysis, we found that it is appropriate to use 3-sigma rule of Gaussian distribution. The property of Gaussian PDF [11] is that only 0.001% of values lies beyond three times the standard deviation towards both sides of the mean value. Therefore we assumed that the value of a Gaussian variable lies within the above specified range, which gives us maximum interference power in center band, \( P_{l_{\text{max,c}}} \) in dB is given as \( P_{l_{\text{max,c}}} = P_{l_{\text{mean,c}}} + 3\sigma_{\text{Pc}} \), where \( P_{l_{\text{mean,c}}} \) and \( \sigma_{\text{Pc}} \) are the mean and standard deviation of the received power in center band. Since SINR is fixed as \( \gamma_{1h} \), it is known that the received power is proportional to the interference power. Hence at a location, when the interference power is maximum, the received power \( (P_{r_{\text{max}}}) \) also attains its maximum value. Therefore, this can be written as \( P_{r_{\text{max}}} = P_{l_{\text{max,c}}} + \gamma_{1h} \). Now the maximum reuse three SINR at the location, \( \gamma_{\text{max,e}} \) is derived as

\[
\gamma_{\text{max,e}} = P_{r_{\text{max}}} - P_{r_{\text{min}}},
\]

where \( P_{r_{\text{min}}} \), the minimum reuse three power at the point is given as \( P_{r_{\text{min}}} = P_{l_{\text{mean,e}}} - 3\sigma_{\text{Pc}} \). This \( \gamma_{\text{max,e}} \) is substituted in (6) to evaluate average bandwidth of edge users. The above derived approach is used while evaluating the average bandwidth requirement of users in a cell.

Fig. 3 shows the analytical upper limit of edge band SINR variation while changing \( \gamma_{1h} \). It follows the same trend as obtained through simulations (99.8 percentile) and simulated upper limit is always below the theoretical upper bound. The close agreement between the theory and simulation results shows that the proposed method is a good approximation and indicates the correctness of the model.

Now, to evaluate the bandwidth requirement, the cell that has two bands, is divided into number of rings or classes based on the SINR of users. The users with similar SINR condition are grouped into one user class [9]. Each user class is associated with certain number of carriers required to make a call. The average number of subcarriers required per user for center and edge band are given as

\[
N_{sc,c} = E_A \left[ \frac{\sum_{i=1}^{N} F_c(\gamma_{i+1}) - F_c(\gamma_i)}{P_{A_c}} N_i \right], \quad \text{and} \quad (8)
\]

\[
N_{sc,e} = E_A \left[ \frac{\sum_{i=1}^{N} (F_e(\gamma_{i+1}) - F_e(\gamma_i)) N_i}{\sum_{i=1}^{N} F_e(\gamma_i)} \right], \quad (9)
\]

where \( N \) is the total (maximum) number of rings in terms of SINR in a cell, \( P_{A_c} \) is the area averaged probability of selecting the center band, \( N_i \) is the number of subcarriers in \( i^{th} \) ring whose boundaries are estimated in terms of SINR at \((i+1)^{th} \) ring \( \gamma_{i+1} \), and SINR at \( i^{th} \) ring \( \gamma_i \). The values of \( N_i, \gamma_{i+1} \) and \( \gamma_i \) are obtained from (4). The numerator term \( F_c(\gamma_{i+1}) - F_c(\gamma_i) \) in (8) is the probability of a user to be in \( i^{th} \) ring of center band, and \( F_e(\gamma) \) is given as

\[
P_c(\gamma_{\text{u,c}}(r, \theta) > \gamma_{1h}) = \frac{1}{2} - \frac{1}{2} \text{erf} \left( \frac{\gamma_{1h} - \overline{\gamma}_{\text{u,c}}(r, \theta)}{\sigma_{\gamma}} \right) = F_c(\gamma), \quad (10)
\]

\[
P_e(\gamma_{\text{u,e}}(r, \theta) < \gamma_{1h}) = \frac{1}{2} + \frac{1}{2} \text{erf} \left( \frac{\gamma_{1h} - \overline{\gamma}_{\text{u,e}}(r, \theta)}{\sigma_{\gamma}} \right) = F_e(\gamma) \quad (11)
\]

The probability of a user to be at center band can be seen from Fig. 4. It is observed that as \( \gamma_{1h} \) increases the probability of an user to be at center band is low, or the probability of an user to be at edge band is high. In (9), \( F_c(\gamma_{i+1}) - F_c(\gamma_i) \) is the probability of a user to be in \( i^{th} \) ring or \( i^{th} \) user class of edge band. Presently no closed form expression is available for \( F_e(\gamma) \), therefore it is obtained semi-analytically as per the following steps: (i) Initially, the users are dropped at number of locations (number of locations-number of drops x number
of users per drop). (ii) We obtain their SINR values. (iii) A user is considered as center band user by satisfying the condition, if the user SINR is greater than the threshold, otherwise it is edge band user. (iv) The probability of selecting edge band is obtained by finding the probability of a user SINR is less than $\gamma_{th}$, which gives $F_{e}(\gamma)$. This $F_{e}(\gamma)$ obtained through these steps is used in (9) while evaluating the number of subcarriers in the edge band in (8). By implementing this approach, the average bandwidth required by a user in a band $b$ is

$$\bar{B}_{b} = \bar{N}_{scb} \Delta f_{sc},$$

(12)

where $\bar{N}_{scb}$ is average number of subcarriers required by a user in band $b$.

IV. ANALYTICAL AND SIMULATION RESULTS DISCUSSION

This section presents numerical and simulation results. The simulation parameters are given in Table I. Fig. 5 shows the average bandwidth required by cell center and cell edge users at different $\gamma_{th}$. It is seen that the analytical result almost matches with the simulations at all values of $\gamma_{th}$. This close agreement shows that the analytical approach is a good approximation. However, by comparing the bandwidth requirement of users, edge users require more amount of bandwidth when compared to center users. For example, at $\gamma_{th}$ of 0 dB, the bandwidth required by a center user is 14 KHz whereas the same required by edge user is 70 KHz. This difference in usage of bandwidth will influence the total number of users supported in a cell. However, a suitable choice of $\gamma_{th}$ is vital in the proper operation of flexible reuse cellular networks and that the choice is heavily dependent on the traffic type and objective of interest. If the objective is to improve both the cell edge and total cell performance, then one has to choose the value which improves both. In practice, the choice of selecting the parameter will be left to the system designer based on his/her requirements.

V. CONCLUSION

In this work we have proposed the analytical approach to evaluate the average bandwidth required by cell center and cell edge users in FFR scheme for real time traffic. We have presented the analytical approach of evaluating the upper limit of SINR of cell edge users. The close agreement between the analytical and simulation curves indicates that the method can be a good approximation for bandwidth required by the users in a cellular environment. By this resource evaluation approach, the number of users supported in a cell can be determined.

REFERENCES


